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mineral processing, operation modeling, simulation, prediction accuracy

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STUDIES ON ADDITIVE PROPERTIES OF SOME PROCESSING OPERATIONS

The accuracy of prediction of mineral processing operation by partial effects summation is analyzed. Methodological and practical aspects of the problem are discussed using laboratory studies of selected comminution and classifying operations as examples. Laboratory experiments show that the effects of examined mineral processing operation depend on simultaneously running processes of classification and comminution. The influence rate of interaction between both processes on the final results is significant, but in some cases it may be neglected. The obtained results have preliminary character and needed a further verification.

1. INTRODUCTION

1.1. Aristotle said that the total is greater that the sum of parts. It may be true in some systems, but seems to be not true in the case of mineral processing operations. This is the subject of the present paper dealing with selected comminution or classifying operations.

When the whole is equal to the sum of its parts, then such a property will be called additiveness. In mineral processing, the problem of additiveness arise in process modeling and simulation or more precisely, always there, where an attempt is made to predict the results of a given operation through analysis of its elements. A good example for this question is the comminution operation. Its capacity or product size greatly

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depends on the distribution of feed particle size. For prediction of the operation results the following model can be used (Whiten 1974, Lynch 1977, Malewski 1981):

$$\mathbf{p} = \mathbf{B} \cdot \mathbf{f} \tag{1}$$

Where: $\mathbf{p} = \left| f(d_i) \right|_{i=1.2..n}$ – is a vector of the operation product size

$\mathbf{B} = \left| f(d_i, D_j)_{i,j=1,2..n} \right|$ – is a breakage matrix of d_i-size of product obtained from D_j-size of feed

$\mathbf{f} = \left| f(D_j) \right|_{j=1.2..n}$ – is a vector of the operation feed size.

Similar experiments and calculations can be done for prediction of either throughput or efficiency of the operation, i.e.

$$Q = \mathbf{G} \cdot \mathbf{f} \tag{2}$$

where Q is a throughput of an operation (single number), $\mathbf{G}$ is vector of an device throughput dependent on feed particles size distribution $\mathbf{f}$.

Thus, if a set of feed particles is divided into fractions, then the effect of the operation one can compute by summing up the comminution products of the individual fractions. However, there may be some doubts whether such a procedure is correct, since the result of independent processing of narrow fractions of particles does not consider the effect of particles of other sizes which are simultaneously processed. Thus, if result (1) is compared to corresponding experimental result $\mathbf{p}^*$, obtained from comminution of a wide class feed particles, then the difference between the comparable quantities indicates the evidence of the nonadditiveness of the analyzed process. This is the subject of our experimental studies made on selected processing operations.

1.2. Relative or absolute differences between the results of operations obtained from calculations based on a summation of parts and those obtained in one operation for a wide class of feed particles, can be used as a measure of operation nonadditiveness. Thus, if result of an operation is given by a number, then a ratio of numbers can express the measure of nonadditiveness, e.g.

$$c_Q = \frac{Q^*}{Q} \tag{3}$$

where: Q^* and Q – are the real and computed effect of an operation, respectively.

If the result of an operation is expressed by a function, as in the case of particle size distribution of operation products, then the differences between functions will be expressed by a mean-square deviation error of the compared functions

$$\Delta P = \left\{ \frac{1}{n} \sum_{i}^{n} [p(d^{*}_{ii}) - p(d_i)]^2 \right\}^{1/2} \tag{4}$$

where n is the number of compared points (particle fractions).

1.3. NOTES

A simplified notation of quantities will be used, i.e.: p_i, f_j, f_{ij} instead of $p(d_i)$, $f(D_j)$, $f(d_i,D_j)$, while the subscripts mean i-fraction number of product particles, j-fraction number of feed particles.

In the matrix notation, the first subscript refers to the row and the second one to the column (vector) of the matrix. Letter p or f always stands for fraction frequency (yield or relative share of a particle class) and F for cumulative distribution of particle size.

All experiments were performed twice: 1) for narrow fractions of the feed particles, and 2) for a wide class of feed particles of size distribution represented by Gaudin-Schuhmann equations with parameter m

$$F(D) = \left(\frac{D}{D_{\max}} \right)^m, \ m = (0.5,\ 1{,}2). \tag{5}$$

EXPERIMENTS

Granite sample was crushed in a laboratory single toggle jaw crusher (Frankiewicz et al., 1978). The throughput and grain size distribution of the crushing product were measured after feeding the device with six narrow j-fractions of the feed particles The same experiment was repeated k-times with a wide class of particle size distribution prepared accordingly to given parameter m_k of equation (5). Detailed data for the device and material used for crushing are shown and Fig.1. and the Table below

$$Q_k = Q_j \cdot f_{jk} =$$

0-8	8-12	12-16	16-25	25-40	40-60
1.412	0.748	0.698	0.632	0.560	0.460

x

m=0.5	m=1	m=2
0.365	0.133	0.018
0.082	0.067	0.022
0.070	0.067	0.031
0.129	0.150	0.103
0.171	0.250	0.271
0.183	0.333	0.555

=

= Qk: =	0.887	0.673	0.536
Qk*:	0.698	0.614	0.535
c=Q*/Q:	0.787	0.913	0.999

Note: The orthogonal matrix refers to the feed particle size distribution (columns) in the successive k=(1,2,3) experiments.

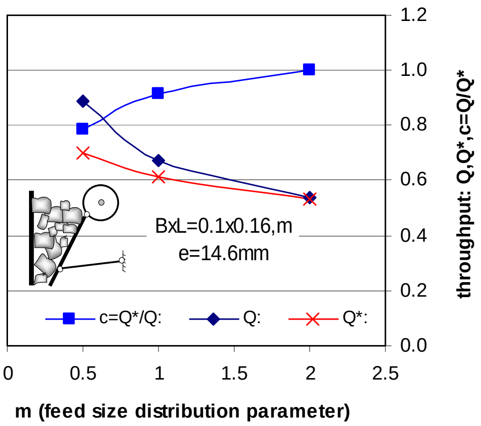


Fig. 1.Throughput of the crusher, kg/s: Q – computed, Q^* – from experiments vs. feed size distribution represented by parameter

2.2. Crushing additive property in the case of product size distribution was studied by examining results of crushing granite rock particles from 0 to 60 mm in size with laboratory 2-toggle jaw crusher which parameters are shown in Fig. 2. Feed size distribution was described by *GS* equation (5) with the parameters m = (0.5, 1.0, 2.0). The set of feed or product particles was classified into following fractions: 2–5, 5–8, 8–10, 10–12, 12–16, 16–25, 25–32, and 32–40 mm. The experiments and calculations provided the following results:

$$\left|p_{ik}\right| = \left|f_{ij}\right| \cdot \left|f_{jk}\right| =$$

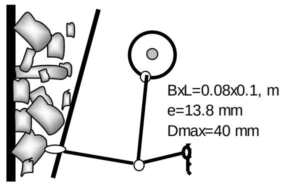
d\D	0-2	2-5	5-8	8-10	10-12	12-16	16-20	20-25	25-32	32-40		m=0.5	m=1	m=2
0-2	1	0.043	0.005	0.009	0.008	0.05	0.057	0.062	0.071	0.095		0.224	0.05	0.003
2-5	0	0.957	0.054	0.011	0.008	0.034	0.017	0.03	0.035	0.04		0.13	**0.075**	0.013
5-8	0	0	0.941	0.158	0.047	0.125	0.057	0.062	0.082	0.093		0.094	0.075	0.024
8-10	0	0	0	0.822	0.089	0.108	0.036	0.043	0.048	0.051		0.053	0.05	0.022
10-12	0	0	0	0	0.848	0.325	0.081	0.083	0.09	0.105	*	0.048	0.05	0.027
12-16	0	0	0	0	0	0.358	0.205	0.239	0.213	0.202		0.085	0.1	0.07
16-20	o	0	0	0	0	0	0.545	0.313	0.228	0.214		0.075	0.1	0.09
20-25	0	0	0	0	0	0	0	0.168	0.206	0.156		0.083	0.125	0.141
25-32	0	0	0	0	0	0	0	0	0.027	0.046		0.104	0.175	0.249
32-40	0	0	0	0	0	0	0	0	0	0		0.106	0.2	0.36

p =

m=0.5	m=1	m=2
0.262	0.104	0.073
0.145	0.100	0.045
0.138	0.140	0.104
0.074	0.084	0.068
0.102	0.130	0.125
0.109	0.164	0.203
0.113	0.176	0.227
0.052	0.088	0.131
0.008	0.014	0.023
0.000	0.000	0.000

p*=

m=0.5	m=1	m=2
0.288	0.083	0.08
0.17	0.073	0.04
0.131	0.098	0.089
0.062	0.097	0.057
0.068	0.098	0.108
0.145	0.217	0.217
0.099	0.208	0.248
0.023	0.112	0.135
0.014	0.014	0.036
0	0	0



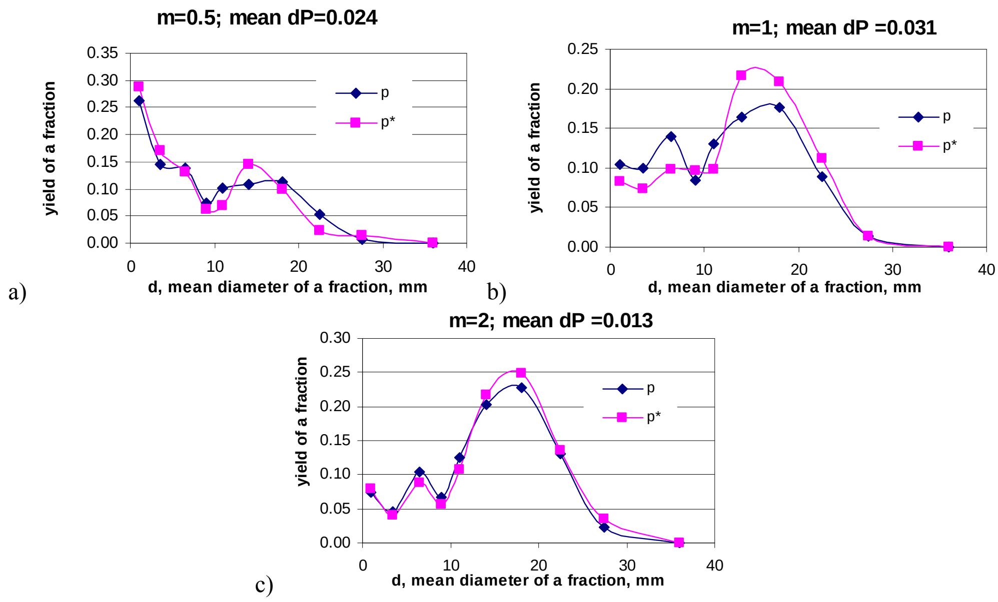


Fig.2. Double-toggle laboratory crusher. Comparison of product size distributions: computed $\mathbf{p}_k$ and observed $\mathbf{p}_k^*$ for given m_k parameter. *Mean dP* as in formula (4)

2.3. The next experiments regard silicate rock comminution in a laboratory air-jet mill. Experimental data published by (Gogala 1979) were used. The following particle size fractions were distinguished: 0–0.2, 0.2–0.5, 0.5–0.9, 0.9–1.4, and 1.4–2 mm. Results of the experiments, particle size distribution of feed and the results of the calculations are presented in the following matrices:

$$[p_{ik}]=[f_{ij}]\cdot[f_{jk}]=$$

	0-0.2	0.2-0.5	0.5-0.9	0.9-1.4	1.4-2
0-0.2	1.000	0.240	0.108	0.079	0.041
0.2-0.5	0.000	0.750	0.226	0.115	0.061
0.5-0.9	0.000	0.000	0.666	0.280	0.078
0.9-1.4	0.000	0.000	0.000	0.524	0.199
1.4-2	0.000	0.000	0.000	0.000	0.612

x

m=0.5	m=1	m=2
0.000	0.000	0.000
0.700	0.250	0.100
0.100	0.250	0.100
0.100	0.250	0.100
0.100	0.250	0.700

p=

m=0.5	m=1	m=2
0.191	0.117	0.071
0.565	0.288	0.152
0.102	0.256	0.149
0.072	0.181	0.192
0.061	0.153	0.428

p*=

m=0.5	m=1	m=2
0.172	0.123	0.055
0.525	0.308	0.146
0.128	0.254	0.160
0.093	0.165	0.238
0.081	0.150	0.402

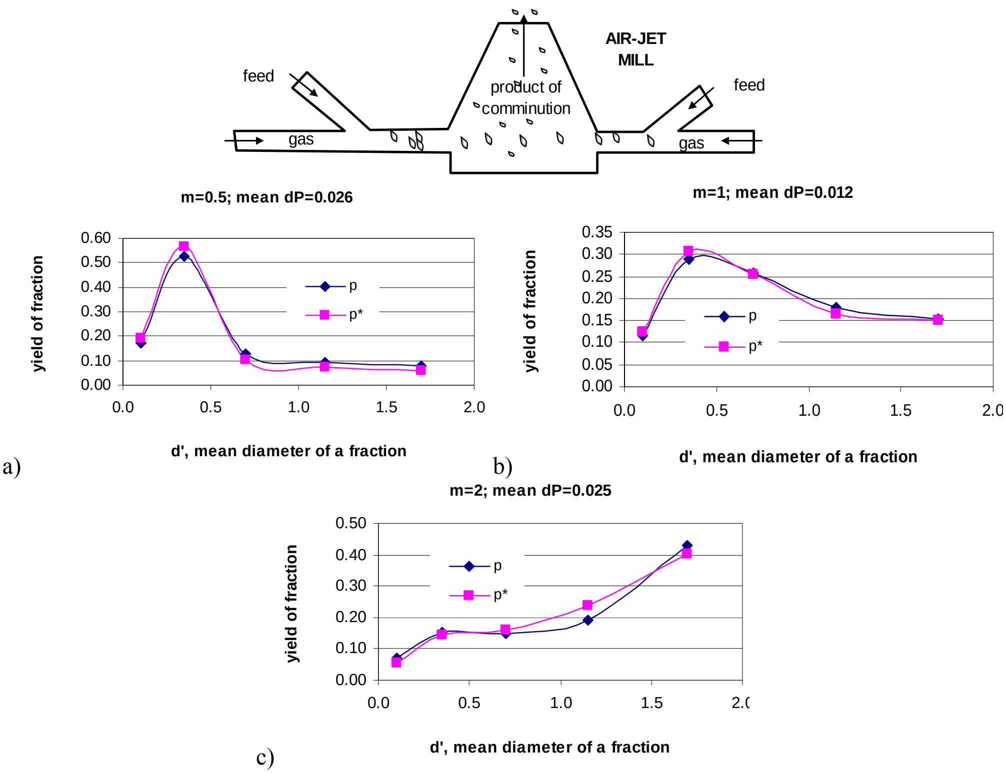


Fig. 3. Laboratory air-jet mill. Comparison of product size distributions: computed p_k and observed $\mathbf{p}_k^*$ for given m_k parameter, *mean dP* as in (4)

It should be noted that the feed used for the experiment was devoid of the fraction of the finest particles and particle size distribution was not approximated precisely by the GGS equation.

2.4. Additiveness of the batch grinding process was also studied using a laboratory ball mill with cyclic discharge. A sample of quartz was ground for 4 minutes. Calculations and comparison of results were conducted in the similar way as in previous examples. The following 6 particle fractions were analyzed: 0–0.43, 0.43–0.6, 0.6–1.02, 1.02–1.5, 1.5–2, and 2–2.5 mm.

A wide class of feed particles was prepared from particle size distribution with m_k= (0.5, 1.0, 2.0), with the exception of the 0–0.43 mm fraction. The results of the experiments and calculations are as follows:

$$[p_{ik}]=[f_{ij}]\cdot[f_{jk}]=$$

	0-0.43	0.43-0.6	0.6-1.02	1.02-1.5	1.5-2	2-2.5
0-0.43	1	0.463	0.301	0.169	0.104	0.076
0.43-0.6	0	0.537	0.224	0.063	0.025	0.012
0.6-1.02	0	0	0.475	0.205	0.065	0.029
1.02-1.5	0	0	0	0.562	0.251	0.094
1.5-2	0	0	0	0	0.556	0.112
2-2.5	0	0	0	0	0	0.677

x

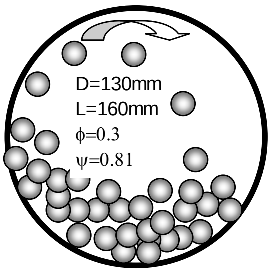
m=0.5	m=1	m=2
0.000	0.000	0.000
0.128	0.082	0.029
0.255	0.203	0.111
0.233	0.231	0.200
0.203	0.242	0.289
0.181	0.242	0.371

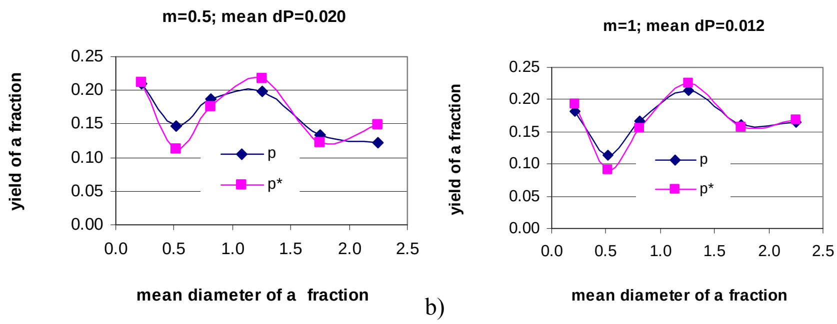
p=

m=0.5	m=1	m=2
0.210	0.182	0.139
0.148	0.113	0.065
0.187	0.167	0.123
0.199	0.213	0.220
0.133	0.162	0.202
0.123	0.164	0.251

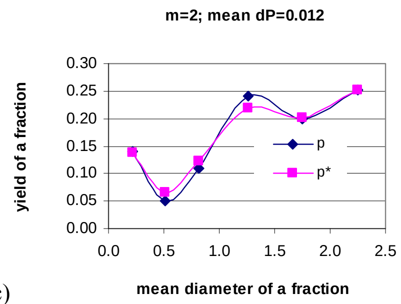
p*=

m=0.5	m=1	m=2
0.212	0.193	0.141
0.113	0.091	0.050
0.175	0.155	0.109
0.217	0.225	0.240
0.122	0.158	0.200
0.148	0.168	0.252





a) b)



c)

Fig.4. Laboratory batch mill. Comparison of product size distributions: computed $\mathbf{p}_k$ and observed $\mathbf{p}_k^*$ for given m_k parameter. Mean dP as in (4)

2.5. Basalt particles were classified by means of a laboratory vibrating screen of the following parameters: B×L = 0.22×0.89 m, aperture #d = 10 mm (Kaniak 1978). The length of the screen was divided into n = 10 sections of $1/n$ length. The efficiency of screening of the set of particles in the $L_n = \sum_1^n l_n$ section was examined. In sequential experiments the screen was loaded with narrow fractions of particles and with the feed of a given particle size distribution described by the *GS* equation with parameter m_k = (0.5, 1,2). The efficiency of screening (particle fraction recovery) of narrow size fraction was denoted by $\varepsilon(L_n, D_j)$ and that of wide size fraction by ε (in short: $\varepsilon, \varepsilon_{nj}$).

Particles size fractions, screen sections length and calculations of fraction recovery are as follows:

ε x f =

Ln, m	2-4	4-6.3	6.3-8	8-10	10-16
0.08	0.271	0.149	0.044	0.009	0
0.17	0.506	0.292	0.086	0.016	0
0.26	0.681	0.409	0.131	0.023	0
0.35	0.804	0.487	0.167	0.029	0
0.44	0.906	0.559	0.206	0.039	0
0.53	0.982	0.632	0.253	0.052	0
0.62	0.999	0.702	0.299	0.064	0
0.71	0.999	0.785	0.365	0.086	0
0.8	1	0.851	0.418	0.104	0
0.89	1	0.915	0.474	0.122	0

x

m=0.5	m=1	m=2
0.231	0.143	0.048
0.200	0.171	0.099
0.123	0.114	0.091
0.123	0.143	0.142
0.323	0.429	0.620

Ln	ε= m=0.5	m=1	m=2	ε*= m=0.5	m=1	m=2
0.08	0.099	0.071	0.033	0.153	0.125	0.066
0.17	0.188	0.134	0.063	0.261	0.207	0.111
0.26	0.258	0.186	0.088	0.336	0.263	0.142
0.35	0.307	0.221	0.106	0.379	0.296	0.161
0.44	0.351	0.254	0.123	0.417	0.324	0.177
0.53	0.391	0.285	0.140	0.447	0.346	0.193
0.62	0.416	0.306	0.154	0.472	0.364	0.206
0.71	0.443	0.331	0.171	0.498	0.384	0.223
0.8	0.465	0.351	0.185	0.517	0.400	0.235
0.89	0.487	0.371	0.199	0.536	0.417	0.246

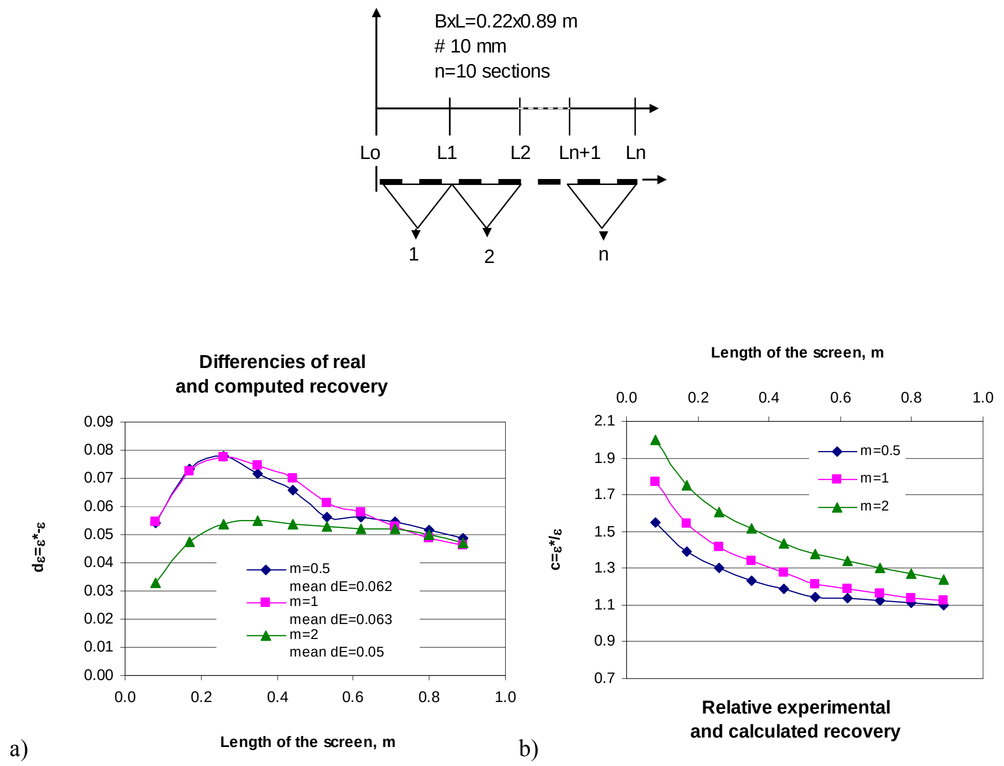
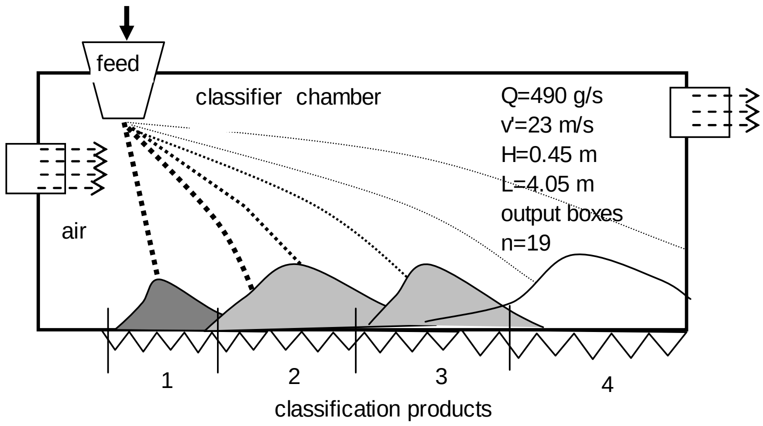


Fig. 5. Nonadditiveness effect of screening depend on feed size distribution and distance from beginning of the screen: $\varepsilon(L)$ – recovery predicted, $\varepsilon(L)^*$ – observed, c – rate of non-additiveness

2.6. The next example is the size classification of particles with a horizontal laboratory air classifier (Kaczkowski 1978). Four products were obtained by dividing the zone of classification into four sections. The fractions of particles in the feed and in the products, and the lengths of classification zones were as follows:



The wide size fraction of particles were approximated by the GS equation with parameters m:= 0.5, 1.0, 2.0; D_{max} = 20 mm. The results of the studies and calculations are as follows (only final results are presented):

m=0.5

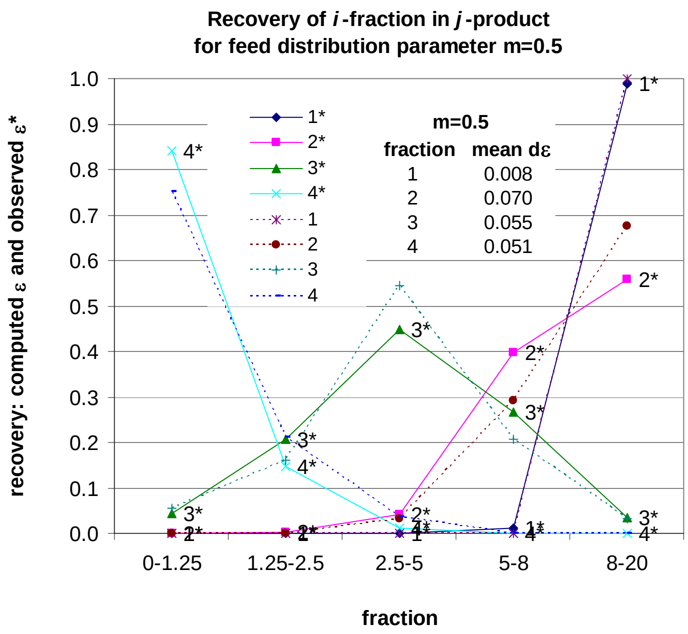
		1	2	3	4			1*	2*	3*	4*
	0-1.25	0.000	0.000	0.054	0.751		0-1.25	0.000	0.000	0.043	0.841
	1.25-2.5	0.000	0.000	0.161	0.212		1.25-2.5	0.000	0.002	0.207	0.147
ε=	2.5-5	0.000	0.032	0.545	0.037	ε*=	2.5-5	0.000	0.041	0.449	0.012
	5-8	0.000	0.293	0.207	0.000		5-8	0.012	0.397	0.267	0.000
	8-20	1.000	0.676	0.032	0.000		8-20	0.988	0.559	0.034	0.000

m=1

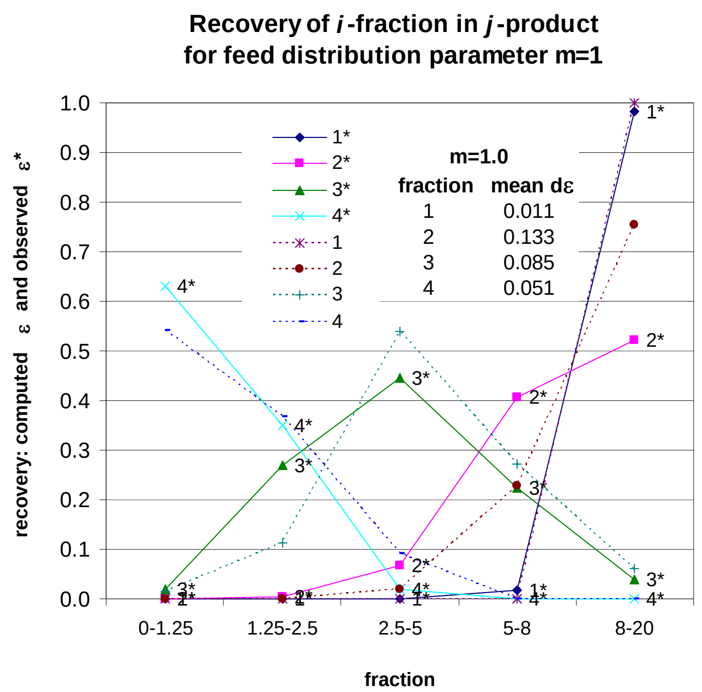
		1	2	3	4			1*	2*	3*	4*
	0-1.25	0.000	0.000	0.016	0.542		0-1.25	0.000	0.000	0.019	0.630
	1.25-2.5	0.000	0.000	0.113	0.368		1.25-2.5	0.000	0.004	0.270	0.350
ε=	2.5-5	0.000	0.019	0.539	0.091	ε*=	2.5-5	0.000	0.067	0.447	0.020
	5-8	0.000	0.228	0.272	0.000		5-8	0.017	0.407	0.225	0.000
	8-20	1.000	0.754	0.060	0.000		8-20	0.983	0.522	0.039	0.000

m=2

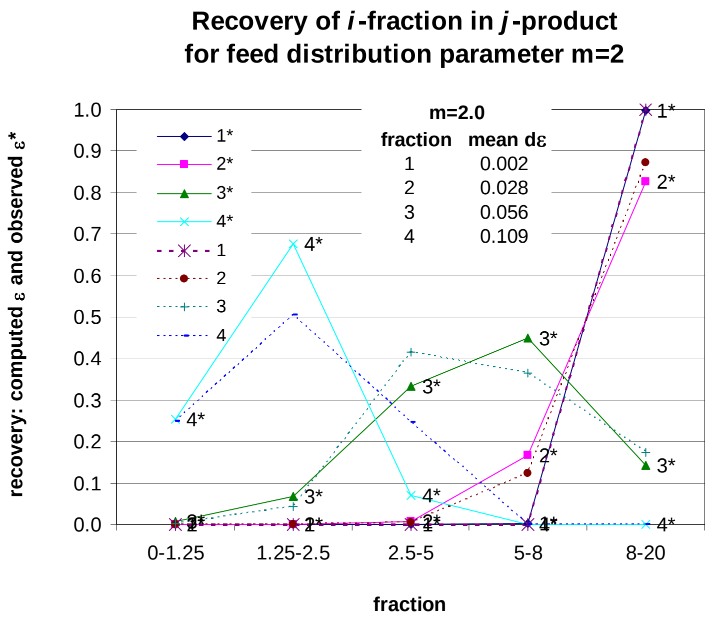
		1	2	3	4			1*	2*	3*	4*
	0-1.25	0.000	0.000	0.002	0.248		0-1.25	0.000	0.000	0.008	0.254
	1.25-2.5	0.000	0.000	0.044	0.506		1.25-2.5	0.000	0.000	0.068	0.676
ε=	2.5-5	0.000	0.006	0.416	0.246	ε*=	2.5-5	0.001	0.008	0.332	0.070
	5-8	0.000	0.123	0.364	0.000		5-8	0.002	0.166	0.449	0.000
	8-20	1.000	0.871	0.174	0.000		8-20	0.997	0.826	0.142	0.000


Recovery of i-fraction in j-product
for feed distribution parameter m=0.5
recovery: computed ε and observed ε*
fraction
0-1.25
1.25-2.5
2.5-5
5-8
8-20
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1*
2*
3*
4*
1
2
3
4
m=0.5
fraction mean dε
1 0.008
2 0.070
3 0.055
4 0.051

a)


Recovery of i-fraction in j-product
for feed distribution parameter m=1
recovery: computed ε and observed ε*
fraction
0-1.25
1.25-2.5
2.5-5
5-8
8-20
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1*
2*
3*
4*
1
2
3
4
m=1.0
fraction mean dε
1 0.011
2 0.133
3 0.085
4 0.051

b)



c)

Fig. 6. Results of recovery of *i*-fraction to the *j*-product of screening depends on the feed size distribution and distance along classifier chamber: ε-computed, ε *-observed.

FINAL REMARKS AND CONCLUSIONS

The analysis of experimental results reveals that non-additiveness is not evident in each process and each operation. The difference between product particle size distribution shown in the figures sometimes seems to have random character. However, this is misleading due to the interactions of particles during processing, and because of loss of energy supplied to each particle leading to resistance of the environment (others particles). Such a phenomenon is always observed during either screening process when each particle must penetrate the layer of other particles being screened or during comminution where the energy supplied to each particle is partly absorbed by the environment. In the performed experiments one can observe some differences between calculated and observed results of either comminution or size classifying. This problem should always be taken into account in operation modeling, especially in the case of wide size fractions.

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STUDIUM ADDYTYWNOŚCI PROCESÓW PRZERÓBCZYCH

W artykule przedstawiono zagadnienie nieaddytywności procesów przeróbczych, jakie występuje w realnych procesach i które powinno być brane pod uwagę w przypadku konstruowania matematycznych modeli efektywności operacji uwzględniających wpływ składu ziarnowego. W warunkach laboratoryjnych wykonano testy i pomiary wydajności i składu ziarnowego na sześciu różnych maszynach: kruszarkach, młynach, przesiewaczu wibracyjnym i klasyfikatorze powietrznym. Badania pokazały, że w przypadku wydajności maszyn rozdrabiających i sprawności przesiewania modele tych operacji powinny uwzględniać nieaddytywność procesu.. Natomiast różnice (*dP*) w składzie ziarnowym produktów rozdrabiania były niewielkie i nie przekraczały trzech procent